Introduction

What is Linear Algebra about?

Linear Algebra is a branch of mathematics which emerged years back and was one of the pioneer branches of mathematics Though, initially it started with solving of the simple linear equation ax + b = 0, but later on it diversified due its vivid application in various fields The basic idea behind finding the solution of a system of linear equations is by elimination of variables

The main reason behind the popularity of algebraic mathematics is following generic solution which most of the problems follow. E.g. start from a problem in geometry, translate the problem in the language of algebra, solve the resulting algebra problem using the algebraic tools and finally transport the solution back to geometry.

History

Matrix, a set of numbers arranged in rows and columns so as to form a rectangular array. The numbers are called the elements, or entries, of the matrix. Matrices have wide applications in engineering, physics, economics, and statistics as well as in various branches of mathematics. Historically, it was not the matrix but a certain number associated with a square array of numbers called the determinant that was first recognized. Only gradually did the idea of the matrix as an algebraic entity emerge. That such an arrangement could be taken as an autonomous mathematical object, subject to special rules that allow for manipulation like ordinary numbers, was first conceived in the 1850s by Cayley and his good friend the attorney and mathematician James Joseph Sylvester.

What do you learn here?

Key Terms:

1. Matrix
2. Vector
3. Submatrix
4. Square matrix
5. Equal matrices
6. Zero matrix
7. Identity matrix
8. Diagonal matrix
9. Upper triangular matrix
10. Lower triangular matrix
11. Tri-diagonal matrix
12. Diagonally dominant matrix

13. Addition of matrices
14. Subtraction of matrices
15. Multiplication of matrices
16. Scalar Product of matrices
17. Linear Combination
18. Rules of Binary Matrix Operation
19. Transpose

20. Symmetric Matrix
21. Skew-Symmetric Matrix
22. Trace of Matrix
23. Determinant
24. Consistent system
25. Inconsistent
26. Infinite solutions
27. Unique solution
28. Rank
29. Inverse

30. Eigenvalue
31. Eigenvectors
32. Power method

What does a matrix look like?

Matrices are everywhere. Look at the matrix below about the sale of jeans in a Departmental Store – given by quarter and make of jeans.

	Q1	Q2	Q3	Q4
Levis	25	20	3	2
Newport	5	10	15	25
Pepe	6	16	7	27

If one wants to know how many *Pepe* jeans were sold in *Quarter 4*, we go along the row *Pepe* and column *Q4* and find that it is 27.

So what is a matrix?

A *matrix* is a rectangular array of elements. The elements can be symbolic expressions or numbers. Matrix $[A]$ is denoted by

$$[A]=\begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} \end{bmatrix}$$

Row i of $[A]$ has n elements and is

$$\begin{bmatrix} a_{i1} & a_{i2} \ldots . a_{in} \end{bmatrix}$$

and column j of $[A]$ has m elements and is

$$\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$$

Each matrix has rows and columns and this defines the size of the matrix. If a matrix $[A]$ has m rows and n columns, the size of the matrix is denoted by $m \times n$. The matrix $[A]$ may also be denoted by $[A]_{m\times n}$ to show that $[A]$ is a matrix with m rows and n columns.

Each entry in the matrix is called the entry or element of the matrix and is denoted by a_{ij} where i is the row number and j is the column number of the element.

The matrix for the jeans sales example could be denoted by the matrix $[A]$ as

$$[A] = \begin{bmatrix} 25 & 20 & 3 & 2 \\ 5 & 10 & 15 & 25 \\ 6 & 16 & 7 & 27 \end{bmatrix}.$$

There are 3 rows and 4 columns, so the size of the matrix is 3×4. In the above $[A]$ matrix, $a_{34} = 27$.

What are the special types of matrices?

Vector: A vector is a matrix that has only one row or one column. There are two types of vectors – row vectors and column vectors.

Row Vector:

If a matrix $[B]$ has one row, it is called a row vector $[B] = [b_1 \; b_2 \ldots\ldots b_n]$ and n is the dimension of the row vector.

Example

Give an example of a row vector.

Solution

$$[B] = [25 \;\; 20 \;\; 3 \;\; 2 \;\; 0]$$

is an example of a row vector of dimension 5.

Column vector:

If a matrix $[C]$ has one column, it is called a column vector

$$[C] = \begin{bmatrix} c_1 \\ \vdots \\ \vdots \\ c_m \end{bmatrix}$$

and m is the dimension of the vector.

Example

Give an example of a column vector.

Solution

$$[C] = \begin{bmatrix} 25 \\ 5 \\ 6 \end{bmatrix}$$

is an example of a column vector of dimension 3.

Submatrix:

If some row(s) or/and column(s) of a matrix $[A]$ are deleted (no rows or columns may be deleted), the remaining matrix is called a submatrix of $[A]$.

Example

Find some of the submatrices of the matrix

$$[A] = \begin{bmatrix} 4 & 6 & 2 \\ 3 & -1 & 2 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 4 & 6 & 2 \\ 3 & -1 & 2 \end{bmatrix}, \begin{bmatrix} 4 & 6 \\ 3 & -1 \end{bmatrix}, \begin{bmatrix} 4 & 6 & 2 \end{bmatrix}, [4], \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

are some of the submatrices of $[A]$. Can you find other submatrices of $[A]$?

Square matrix:

If the number of rows m of a matrix is equal to the number of columns n of a matrix $[A]$, ($m = n$), then $[A]$ is called a square matrix. The entries $a_{11}, a_{22}, \ldots, a_{nn}$ are called the *diagonal elements* of a square matrix. Sometimes the diagonal of the matrix is also called the *principal or main of the matrix*.

Example

Give an example of a square matrix.

Solution

$$[A] = \begin{bmatrix} 25 & 20 & 3 \\ 5 & 10 & 15 \\ 6 & 15 & 7 \end{bmatrix}$$

is a square matrix as it has the same number of rows and columns, that is, 3. The diagonal elements of $[A]$ are $a_{11} = 25, \ a_{22} = 10, \ a_{33} = 7$.

Upper triangular matrix:

A $m \times n$ matrix for which $a_{ij} = 0, \ i > j$ is called an upper triangular matrix. That is, all the elements below the diagonal entries are zero.

Example

Give an example of an upper triangular matrix.

Solution

$$[A] = \begin{bmatrix} 10 & -7 & 0 \\ 0 & -0.001 & 6 \\ 0 & 0 & 15005 \end{bmatrix}$$

is an upper triangular matrix.

Lower triangular matrix:

A $m \times n$ matrix for which $a_{ij} = 0, \ j > i$ is called a lower triangular matrix. That is, all the elements above the diagonal entries are zero.

Example

Give an example of a lower triangular matrix.

Solution

$$[A]=\begin{bmatrix}1 & 0 & 0\\ 0.3 & 1 & 0\\ 0.6 & 2.5 & 1\end{bmatrix}$$

is a lower triangular matrix.

Diagonal matrix:

A square matrix with all non-diagonal elements equal to zero is called a diagonal matrix, that is, only the diagonal entries of the square matrix can be non-zero, ($a_{ij} = 0,\ i \neq j$).

Example

Give examples of a diagonal matrix.

Solution

$$[A]=\begin{bmatrix}3 & 0 & 0\\ 0 & 2.1 & 0\\ 0 & 0 & 0\end{bmatrix}$$

is a diagonal matrix.

Any or all the diagonal entries of a diagonal matrix can be zero. For example

$$[A]=\begin{bmatrix}3 & 0 & 0\\ 0 & 2.1 & 0\\ 0 & 0 & 0\end{bmatrix}$$

is also a diagonal matrix.

Identity matrix:

A diagonal matrix with all diagonal elements equal to one is called an identity matrix, ($a_{ij} = 0,\ i \neq j$ and $a_{ii} = 1$ for all i).

Example

Give an example of an identity matrix.

Solution

$$[A]=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

is an identity matrix.

Zero matrix:

A matrix whose all entries are zero is called a zero matrix, ($a_{ij} = 0$ for all i and j).

Example

Give examples of a zero matrix.

Solution

$$[A] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[\mathrm{B}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

are all examples of a zero matrix.

Tridiagonal matrices:

A tridiagonal matrix is a square matrix in which all elements not on the following are zero - the major diagonal, the diagonal above the major diagonal, and the diagonal below the major diagonal.

Example

Give an example of a tridiagonal matrix.

Solution

$$[A] = \begin{bmatrix} 2 & 4 & 0 & 0 \\ 2 & 3 & 9 & 0 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 3 & 6 \end{bmatrix}$$

is a tridiagonal matrix.

Do non-square matrices have diagonal entries?

Yes, for a $m \times n$ matrix $[A]$, the diagonal entries are $a_{11}, a_{22} ..., a_{k-1,k-1}, a_{kk}$ where $k = \min\{m, n\}$.

Example

What are the diagonal entries of

$$[A]=\begin{bmatrix}3.2 & 5\\ 6 & 7\\ 2.9 & 3.2\\ 5.6 & 7.8\end{bmatrix}$$

Solution

The diagonal elements of $[A]$ are $a_{11}=3.2$ and $a_{22}=7$.

Diagonally Dominant Matrix:

A $n\times n$ square matrix $[A]$ is a diagonally dominant matrix if

$$\left|a_{ii}\right| \geq \sum_{\substack{j=1\\ i\neq j}}^{n} |\, a_{ij}\,| \text{ for all } i=1,2,.....,n \text{ and}$$

$$\left|a_{ii}\right| > \sum_{\substack{j=1\\ i\neq j}}^{n} |\, a_{ij}\,| \text{ for at least one } i\,,$$

that is, for each row, the absolute value of the diagonal element is greater than or equal to the sum of the absolute values of the rest of the elements of that row, and that the inequality is strictly greater than for at least one row. Diagonally dominant matrices are important in ensuring convergence in iterative schemes of solving simultaneous linear equations.

Example

Give examples of diagonally dominant matrices and not diagonally dominant matrices.

Solution

$$[A]=\begin{bmatrix}15 & 6 & 7\\ 2 & -4 & -2\\ 3 & 2 & 6\end{bmatrix}$$

is a diagonally dominant matrix as

$$\left|a_{11}\right|=\left|15\right|=15\geq\left|a_{12}\right|+\left|a_{13}\right|=\left|6\right|+\left|7\right|=13$$

$$\left|a_{22}\right|=\left|-4\right|=4\geq\left|a_{21}\right|+\left|a_{23}\right|=\left|2\right|+\left|-2\right|=4$$

$$\left|a_{33}\right|=\left|6\right|=6\geq\left|a_{31}\right|+\left|a_{32}\right|=\left|3\right|+\left|2\right|=5$$

and for at least one row, that is Rows 1 and 3 in this case, the inequality is a strictly greater than inequality.

$$[B] = \begin{bmatrix} -15 & 6 & 9 \\ 2 & -4 & 2 \\ 3 & -2 & 5.001 \end{bmatrix}$$

is a diagonally dominant matrix as

$$|b_{11}| = |-15| = 15 \geq |b_{12}| + |b_{13}| = |6| + |9| = 15$$

$$|b_{22}| = |-4| = 4 \geq |b_{21}| + |b_{23}| = |2| + |2| = 4$$

$$|b_{33}| = |5.001| = 5.001 \geq |b_{31}| + |b_{32}| = |3| + |-2| = 5$$

The inequalities are satisfied for all rows and it is satisfied strictly greater than for at least one row (in this case it is Row 3).

$$[C] = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

is not diagonally dominant as

$$|c_{22}| = |8| = 8 \leq |c_{21}| + |c_{23}| = |64| + |1| = 65$$

When are two matrices considered to be equal?

Two matrices [*A*] and [*B*] are equal if the size of [*A*] and [*B*] is the same (number of rows and columns are same for [*A*] and [*B*]) and $a_{ij} = b_{ij}$ for all *i* and *j*.

Example

What would make

$$[A] = \begin{bmatrix} 2 & 3 \\ 6 & 7 \end{bmatrix}$$

to be equal to

$$[B] = \begin{bmatrix} b_{11} & 3 \\ 6 & b_{22} \end{bmatrix}$$

Solution

The two matrices $[A]$ and $[B]$ could be equal if $b_{11} = 2$ and $b_{22} = 7$.

How do you add two matrices?

How do you add

Two matrices $[A]$ and $[B]$ can be added only if they are the same size. The addition is then shown as

$$[C] = [A] + [B]$$

where

$c_{ij} = a_{ij} + b_{ij}$

Example

Add the following two matrices.

$$[A] = \begin{bmatrix} 5 & 2 & 3 \\ 1 & 2 & 7 \end{bmatrix} \qquad [B] = \begin{bmatrix} 6 & 7 & -2 \\ 3 & 5 & 19 \end{bmatrix}$$

Solution

$$[C] = [A] + [B]$$

$$= \begin{bmatrix} 5 & 2 & 3 \\ 1 & 2 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 7 & -2 \\ 3 & 5 & 19 \end{bmatrix}$$

$$= \begin{bmatrix} 5+6 & 2+7 & 3-2 \\ 1+3 & 2+5 & 7+19 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 9 & 1 \\ 4 & 7 & 26 \end{bmatrix}$$

Example

ABC Departmental Store has two store locations A and B, and their sales of jeans are given by make (in rows) and quarters (in columns) as shown below.

$$[A] = \begin{bmatrix} 25 & 20 & 3 & 2 \\ 5 & 10 & 15 & 25 \\ 6 & 16 & 7 & 27 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 20 & 5 & 4 & 0 \\ 3 & 6 & 15 & 21 \\ 4 & 1 & 7 & 20 \end{bmatrix}$$

where the rows represent the sale of Levis, Newport and Pepe jeans respectively and the columns represent the quarter number: 1, 2, 3 and 4. What are the total jeans sales for the two locations by make and quarter?

Solution

$$[C] = [A] + [B]$$

$$= \begin{bmatrix} 25 & 20 & 3 & 2 \\ 5 & 10 & 15 & 25 \\ 6 & 16 & 7 & 27 \end{bmatrix} + \begin{bmatrix} 20 & 5 & 4 & 0 \\ 3 & 6 & 15 & 21 \\ 4 & 1 & 7 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} (25+20) & (20+5) & (3+4) & (2+0) \\ (5+3) & (10+6) & (15+15) & (25+21) \\ (6+4) & (16+1) & (7+7) & (27+20) \end{bmatrix}$$

$$= \begin{bmatrix} 45 & 25 & 7 & 2 \\ 8 & 16 & 30 & 46 \\ 10 & 17 & 14 & 47 \end{bmatrix}$$

So if one wants to know the total number of Pepe jeans sold in quarter 4 at the two locations, we would look at Row 3 – Column 4 to give $c_{34} = 47.$

How do you subtract two matrices?

Two matrices $[A]$ and $[B]$ can be subtracted only if they are the same size. The subtraction is then given by

$$[D] = [A] - [B]$$

Where

$$d_{ij} = a_{ij} - b_{ij}$$

Example

Subtract matrix $[B]$ from matrix $[A]$.

$$[A] = \begin{bmatrix} 5 & 2 & 3 \\ 1 & 2 & 7 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 6 & 7 & -2 \\ 3 & 5 & 19 \end{bmatrix}$$

Solution

$$[D] = [A] - [B]$$

$$= \begin{bmatrix} 5 & 2 & 3 \\ 1 & 2 & 7 \end{bmatrix} - \begin{bmatrix} 6 & 7 & -2 \\ 3 & 5 & 19 \end{bmatrix}$$

$$= \begin{bmatrix} (5-6) & (2-7) & (3-(-2)) \\ (1-3) & (2-5) & (7-19) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -5 & 5 \\ -2 & -3 & -12 \end{bmatrix}$$

Example

ABC Departmental Store has two store locations A and B and their sales of jeans are given by make (in rows) and quarters (in columns) as shown below.

$$[A] = \begin{bmatrix} 25 & 20 & 3 & 2 \\ 5 & 10 & 15 & 25 \\ 6 & 16 & 7 & 27 \end{bmatrix}$$

$$[B]=\begin{bmatrix}20 & 5 & 4 & 0\\ 3 & 6 & 15 & 21\\ 4 & 1 & 7 & 20\end{bmatrix}$$

where the rows represent the sale of Levis, Newport and Pepe jeans respectively and the columns represent the quarter number: 1, 2, 3, and 4. How many more jeans did store A sell than store B of each brand in each quarter?

Solution

$$[D]=[A]-[B]$$

$$=\begin{bmatrix}25 & 20 & 3 & 2\\ 5 & 10 & 15 & 25\\ 6 & 16 & 7 & 27\end{bmatrix}-\begin{bmatrix}20 & 5 & 4 & 0\\ 3 & 6 & 15 & 21\\ 4 & 1 & 7 & 20\end{bmatrix}$$

$$=\begin{bmatrix}25-20 & 20-5 & 3-4 & 2-0\\ 5-3 & 10-6 & 15-15 & 25-21\\ 6-4 & 16-1 & 7-7 & 27-20\end{bmatrix}$$

$$=\begin{bmatrix}5 & 15 & -1 & 2\\ 2 & 4 & 0 & 4\\ 2 & 15 & 0 & 7\end{bmatrix}$$

So if you want to know how many more Pepe jeans were sold in quarter 4 in store A than store B, $d_{34}=7$. Note that $d_{13}=-1$ implies that store A sold 1 less Newport jeans than store B in quarter 3.

How do I multiply two matrices?

Two matrices $[A]$ and $[B]$ can be multiplied only if the number of columns of $[A]$ is equal to the number of rows of $[B]$ to give

$$[C]_{m\times n}=[A]_{m\times p}[B]_{p\times n}$$

If $[A]$ is a $m\times p$ matrix and $[B]$ is a $p\times n$ matrix, the resulting matrix $[C]$ is a $m\times n$ matrix.

So how does one calculate the elements of $[C]$ matrix?

$$c_{ij}=\sum_{k=1}^{p}a_{ik}b_{kj}$$

$$=a_{i1}b_{1j}+a_{i2}b_{2j}+\ldots\ldots+a_{ip}b_{pj}$$

for each $i=1,\ 2,\ldots\ldots,m$ and $j=1,\ 2,\ldots\ldots,n$.

To put it in simpler terms, the i^{th} row and j^{th} column of the $[C]$ matrix in $[C]=[A][B]$ is calculated by multiplying the i^{th} row of $[A]$ by the j^{th} column of $[B]$, that is,

$$c_{ij} = \begin{bmatrix} a_{i1} & a_{i2} & \ldots\ldots & a_{ip} \end{bmatrix} \begin{bmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ \vdots \\ b_{pj} \end{bmatrix}$$

$$= a_{i1}\, b_{1j} + a_{i2}\, b_{2j} + \ldots\ldots + a_{ip}\, b_{pj}.$$

$$= \sum_{k=1}^{p} a_{ik} b_{kj}$$

Example

Given

$$[A] = \begin{bmatrix} 5 & 2 & 3 \\ 1 & 2 & 7 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 3 & -2 \\ 5 & -8 \\ 9 & -10 \end{bmatrix}$$

Find

$$[C] = [A][B]$$

Solution

c_{12} can be found by multiplying the first row of $[A]$ by the second column of $[B]$,

$$c_{12} = \begin{bmatrix} 5 & 2 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ -8 \\ -10 \end{bmatrix}$$

$$= (5)(-2) + (2)(-8) + (3)(-10)$$

$$= -56$$

Similarly, one can find the other elements of $[C]$ to give

$$[C] = \begin{bmatrix} 52 & -56 \\ 76 & -88 \end{bmatrix}$$

Example

ABC Departmental Store location *A* and the sales of jeans are given by make (in rows) and quarters (in columns) as shown below

$$[A]=\begin{bmatrix}25 & 20 & 3 & 2\\ 5 & 10 & 15 & 25\\ 6 & 16 & 7 & 27\end{bmatrix}$$

where the rows represent the sale of Levis, Newport and Pepe jeans respectively and the columns represent the quarter number: 1, 2, 3, and 4. Find the per quarter sales of store A if the following are the prices of each jeans.

Levis = $33.25

Newport = $40.19

Pepe = $25.03

Solution

The answer is given by multiplying the price matrix by the quantity of sales of store A. The price matrix is $[33.25 \quad 40.19 \quad 25.03]$, so the per quarter sales of store A would be given by

$$[C]=[33.25 \quad 40.19 \quad 25.03]\begin{bmatrix}25 & 20 & 3 & 2\\ 5 & 10 & 15 & 25\\ 6 & 16 & 7 & 27\end{bmatrix}$$

$$c_{ij}=\sum_{k=1}^{3} a_{ik}b_{kj}$$

$$c_{11}=\sum_{k=1}^{3} a_{1k}b_{k1}$$

$$=a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31}$$

$$=(33.25)(25)+(40.19)(5)+(25.03)(6)$$

$$=\$1182.38$$

Similarly

$$c_{12}=\$1467.38$$
$$c_{13}=\$877.81$$
$$c_{14}=\$1747.06$$

Therefore, each quarter sales of store A in dollars is given by the four columns of the row vector

$$[C]=[1182.38 \quad 1467.38 \quad 877.81 \quad 1747.06]$$

Remember since we are multiplying a 1×3 matrix by a 3×4 matrix, the resulting matrix is a 1×4 matrix.

What is the scalar product of a constant and a matrix?

If $[A]$ is a $n\times n$ matrix and k is a real number, then the scalar product of k and $[A]$ is another $n\times n$ matrix $[B]$, where $b_{ij}=k\,a_{ij}$.

Example

Let

$$[A]=\begin{bmatrix}2.1 & 3 & 2\\ 5 & 1 & 6\end{bmatrix}$$

Find $2[A]$

Solution

$$2[A]=2\begin{bmatrix}2.1 & 3 & 2\\ 5 & 1 & 6\end{bmatrix}$$

$$=\begin{bmatrix}2\times 2.1 & 2\times 3 & 2\times 2\\ 2\times 5 & 2\times 1 & 2\times 6\end{bmatrix} \quad =\begin{bmatrix}4.2 & 6 & 4\\ 10 & 2 & 12\end{bmatrix}$$

What is a linear combination of matrices?

If $[A_1],[A_2],.....[A_p]$ are matrices of the same size and $k_1,k_2,.....,k_p$ are scalars, then

$$k_1[A_1]+k_2[A_2]+........+k_p[A_p]$$

is called a linear combination of $[\mathrm{A}_1],[A_2],.....,[A_p]$

Example

If $[A_1]=\begin{bmatrix}5 & 6 & 2\\ 3 & 2 & 1\end{bmatrix},[A_2]=\begin{bmatrix}2.1 & 3 & 2\\ 5 & 1 & 6\end{bmatrix},[A_3]=\begin{bmatrix}0 & 2.2 & 2\\ 3 & 3.5 & 6\end{bmatrix}$ then find $[A_1]+2[A_2]-0.5[A_3]$

Solution

$$[A_1]+2[A_2]-0.5[A_3]$$

$$=\begin{bmatrix}5 & 6 & 2\\ 3 & 2 & 1\end{bmatrix}+2\begin{bmatrix}2.1 & 3 & 2\\ 5 & 1 & 6\end{bmatrix}-0.5\begin{bmatrix}0 & 2.2 & 2\\ 3 & 3.5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}5 & 6 & 2\\ 3 & 2 & 1\end{bmatrix}+\begin{bmatrix}4.2 & 6 & 4\\ 10 & 2 & 12\end{bmatrix}-\begin{bmatrix}0 & 1.1 & 1\\ 1.5 & 1.75 & 3\end{bmatrix}$$

$$=\begin{bmatrix}9.2 & 10.9 & 5\\ 11.5 & 2.25 & 10\end{bmatrix}$$

What are some of the rules of binary matrix operations?

Commutative law of addition

If $[A]$ and $[B]$ are $m\times n$ matrices, then

$$[A]+[B]=[B]+[A]]$$

Associative law of addition

If [A], [B] and [C] are all $m \times n$ matrices, then

$$[A]+([B]+[C])=([A]+[B])+[C]$$

Associative law of multiplication

If $[A]$, $[B]$ and $[C]$ are $m \times n, n \times p$ and $p \times r$ size matrices, respectively, then

$$[A]([B][C])=([A][B])[C]$$

and the resulting matrix size on both sides of the equation is $m \times r$.

Distributive law

If $[A]$ and $[B]$ are $m \times n$ size matrices, and $[C]$ and $[D]$ are $n \times p$ size matrices

$$[A]([C]+[D])=[A][C]+[A][D]$$

$$([A]+[B])[C]=[A][C]+[B][C]$$

and the resulting matrix size on both sides of the equation is $m \times p$.

Example

Illustrate the associative law of multiplication of matrices using

$$[A]=\begin{bmatrix}1 & 2\\ 3 & 5\\ 0 & 2\end{bmatrix}, \quad [B]=\begin{bmatrix}2 & 5\\ 9 & 6\end{bmatrix}, \quad [C]=\begin{bmatrix}2 & 1\\ 3 & 5\end{bmatrix}$$

Solution

$$[B][C]=$$

$$=\begin{bmatrix}2 & 5\\ 9 & 6\end{bmatrix}\begin{bmatrix}2 & 1\\ 3 & 5\end{bmatrix}$$

$$=\begin{bmatrix}19 & 27\\ 36 & 39\end{bmatrix}$$

$$[A]([B][C])=\begin{bmatrix}1 & 2\\ 3 & 5\\ 0 & 2\end{bmatrix}\begin{bmatrix}19 & 27\\ 36 & 39\end{bmatrix}$$

$$=\begin{bmatrix}91 & 105\\ 237 & 276\\ 72 & 78\end{bmatrix}$$

$$[A][B]=\begin{bmatrix}1 & 2\\ 3 & 5\\ 0 & 2\end{bmatrix}\begin{bmatrix}2 & 5\\ 9 & 6\end{bmatrix}$$