Aptitude for GATE

The GATE syllabus for General Aptitude is as follows:

Verbal Ability: English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.

Numerical Ability: Numerical computation, numerical estimation, numerical reasoning and data interpretation.

However, GATE questions in the past have not necessarily followed this syllabus definition. To prepare for GATE general aptitude, it would require you to prepare all topics under aptitude. However, it would be foolhardy to do so as it is illogical to spend the amount of time required to prepare ‘everything’ vis-à-vis marks in GATE for general aptitude. So we suggest the better option.

We have closely analyzed the GATE question types in General Aptitude. In this book, we will work at two levels. A) Basics that you must know, and b) concepts/tricks that you acquire from solving questions.

Hence, the first part of the book is all the important theory with some practice questions. The second part of the book is a database of questions that cover some question types asked in GATE. After using this book, start solving the online tests.

First go through the concepts and make sure that you know your theory well. Once you have done that, start answering the questions. Wherever you come across a new concept / trick, mark that question as important and come back to it during revision process.

All the best

Chapter 1 # Even and Odd Numbers **Suggested Time : 30 minutes**

The numbers, which leave no remainder when divided by 2 are called even numbers and the numbers, which are not divisible by 2, are called odd numbers. The general form of writing an even number is 2n, and that of an odd number is $2n \pm 1$

Examples of even numbers are 0, 2, 4, 6, – 8, – 12, etc.

Examples of odd numbers are 1, 3, 5, – 7, – 15, etc.

Note:

1. The sum, difference and product of two even numbers are even.
   Example: 6 + 8 = 14, is even.

2. The sum and difference of two odd numbers is an even number.
   Example: 5 + 11 = 16, is even.

3. The product of two odd numbers is an odd number.
   Example: 3 × 7 = 21, is odd.

4. Zero is an even number.

5. (i) $Even^{Even}$ = Even. Example: 2^4 is even
   (ii) $Even^{Odd}$ = Even. Example: 2^3 is even
   (iii) Odd^{Even} = Odd. Example: 3^2 is odd
   (iv) Odd^{Odd} = Odd. Example: 3^3 is odd

Practice Questions A (Important)

1. If x and y are natural numbers, which of the following is definitely odd?
   1] $x^2y^3(x + y)$ 2] $x^3y^2(x + y)$ 3] (2x + 1) (2y + 1) 4] (x + 1)(2y + 1)

2. How many even integers lie between the two values of x (including the extremes), if $x^2 = 100$?
   1] 5 2] 10 3] 11 4] 4

3. If 7x + 3y = 240, which of these statements is true?
   1] x and y are both odd. 2] Sum of x and y is even.
   3] If x is odd, y is even. 4] None of these

4. If x is a positive even integer and y a positive odd integer, then which of the following statements are true?
   i. $[(-1)^x]^y$ is always positive ii. $[(-1)^x]^{-y}$ is always negative
   iii. $[(-1)^x]^{y/2}$ is always negative iv. $(-1)^{x+y}$ is always negative

   1] i and ii only 2] i and iv only 3] ii and iv only 4] i and iii only

5. State whether the following statements are true or false?
   (i) Sum of two odd numbers is never odd.
   (ii) Product of two odd numbers is always odd.
   (iii) Sum of product of two odd numbers and product of two even numbers is sometimes even.
   (iv) Any even power of an odd number is even.
   (v) Product of two prime numbers is never prime.
   (vi) Sum of two prime numbers is never prime.

6. 2x + 3y = 240, where x & y are natural numbers, what can you say about y?
   1] y is odd 2] y is even 3] y is a multiple of 3 4] none of these

Answer Key

1-3 2-3 3-2 4-2 5- False, True, False, False, False, False 6-1

Explanatory Answers

1. $(2x + 1)$ and $(2y + 1)$ are odd.
Odd × Odd is Odd.

2. $x = -10$ or $+10$. Between these integers in the number line, there are 11 even numbers.
$(-10, -8, \ldots 0, 2, 4, \ldots 10)$

3. $7x + 3y = 240$
240 is even. Either odd + odd = even, or even + even = even.
If x is odd, 7x is odd.
∴ 3y has to be odd to make $7x + 3y$ even.
∴ y has to be odd.
Similarly, when x is even, y has to be even.
∴ x and y are either both odd or both even.
∴ $x + y$ is even in both scenarios. Hence [2]

4. i. As x is an even positive integer, $(-1)^x$ is 1 and $[(-1)^x]^y$ is 1. Hence, true.
ii. $(-1)^x$ is 1 and $((-1)^x)^{-y} = \frac{1}{y}$, which is positive. Hence, false.
iii. $(-1)^x$ is 1 and $((-1)^x)^{\frac{y}{2}}$ is positive. Hence, false.
iv. $(-1)^{x+y} = (-1)^{odd} = -1$. Hence, true. Hence [2]

5. (i) is false. (ii) is true. (iii) is false (always odd). (iv) is false (any power of an odd number is odd).
(v) is false (always composite). (vi) is false as $2 + 3 = 5$, which is again prime.

6. $2x + 3y = 240$
Whatever be x, 2x is even.
∴ 3y has to be even to make $2x + 3y$ even (i.e. 240).
∴ y has to be even.

Practice Questions B (Optional)

1. Which of the following is not an even number?
   1] 2 2] 0
   3] – 4 4] All are even numbers

2. The number x ! (x factorial) is definitely
   1] Even 2] Odd
   3] Even if x > 1 4] Even if x is even and off if x is odd

3. 3x + 4y is an even number. Then what can you say about x and y?
   1] x is even, y is odd 2] x and y are both even
   3] x is even, y may be even or odd 4] x is odd, y is even

4. a, x and y are positive integers. $a = x^y$. Then we can say that
   1] a is even 2] a is odd
   3] a is even, if x is even 4] a is odd, if x is even

5. Let x, y and z be distinct integers, x and y are odd and positive, and z is even and positive. Which one of the following statements cannot be true?
   1] $(x-z)^2y$ is even 2] $(x-z)y^2$ is odd
   3] $(x-z)y$ is odd 4] $(x-y)^2z$ is even

6. If n = 1 + x, where x is the product of four consecutive positive integers, then which of the following is/are true?
   A. n is odd B. n is prime C. n is a perfect square
   1] A and C only 2] A and B only
   3] A only 4] None of these

Answer Key

1-4 2-3 3-3 4-3 5-1 6-1

Chapter 2 Tests of Divisibility

Suggested Time : 45 minutes

Divisibility by 2	A number is divisible by 2 when its unit's digit is even. Example: 34, 900, 8616, etc. are divisible by 2 since the last digit is even.
Divisibility by 3	A number is divisible by 3 if the sum of its digits is a multiple of 3. Example: In 5412, the sum of the digits is 5 + 4 + 1 + 2 = 12, which is a multiple of 3. Hence, 5412 is divisible by 3.
Divisibility by 4	A number is divisible by 4 if the number formed by its last two digits is divisible by 4. Example: In 43552, the number formed by its last two digits is 52, which is divisible by 4. Hence, the number is divisible by 4.
Divisibility by 6	A number is divisible by 6 if it is divisible by both 2 and 3. Hence, apply the divisibility test of both 2 and 3. Example: 600, 36, 78, etc. are divisible by 6.
Divisibility by 7	A number is divisible by 7, if the sum of the number of tens in the number and five times the unit digit is divisible by 7. This test may be performed successively to get a smaller multiple of 7. Note that number of tens in 562 is 56 (and not 6) as 562 = 500 + 60 + 2 and 500 has 50 tens while 60 has another 6. Example: 735 is divisible by 7 as 73 + 5(5) = 73 + 25 = 98; 9 + 5(8) = 9 + 40 = 49, which is divisible by 7.
Divisibility by 8	A number is divisible by 8 if the number formed by its last three digits is divisible by 8. Example: In 569048, the last 3 digits of the number is 048, which is divisible by 8. Hence, the number is divisible by 8.
Divisibility by 9	A number is divisible by 9, when the sum of its digits is a multiple of 9. Example: 7281 is divisible by 9 since 7 + 2 + 8 + 1 = 18, which is divisible by 9.
Divisibility by 11	A number is divisible by 11, if the difference of the sum of the digits in odd places and the sum of the digits in even places (starting from units' place) is either 0 or a multiple of 11. For example, 8050314052, is divisible by 11 since the difference between the sum of digits in even places (5 + 4 + 3 + 5 + 8 = 25) and sum of digits in odd places (2 + 0 + 1 + 0 + 0 = 3) is 22 (i.e. 25– 3 = 22), which is divisible by 11.
Divisibility by 12	A number is divisible by 12, when it is divisible by 3 and 4. Example: 180 is divisible by 12, since it is divisible by 3 and 4.
Divisibility by 15	A number is divisible by 15, when it is divisible by both 3 and 5. Example: 1125 is divisible by 15, since it is divisible by 3 and 5.
Divisibility by 25	A number is divisible by 25, when the number formed by the last two digits of the number is divisible by 25. Example: 475350 is divisible by 25 since "50" is divisible by 25.

Note: 0 is divisible by all the numbers.

Practice Questions A (Important)

1. How many of the following number/s is / are divisible by 8 and 11 simultaneously?
(i) 12496 (ii) 414206 (iii) 999000 (iv) 48400
1] 1 2] 2 3] 3 4] 0

2. Which of the following numbers is a prime number?
1] 2181 2] 3585 3] 4587 4] 3931

3. What should you divide 11979 by, to get a perfect cube?
1] 3 2] 9 3] 11 4] 11^3

4. 109521ab is divisible by 22. Which of these is a possible value of a and b?
1] 06 2] 60 3] 54 4] 39

5. How many numbers are there between 100 and 200 (both inclusive), and divisible by 2 or 3?
1] 67 2]68 3] 84 4] 100

6. A 4-digit number is of the form "36ab" where a and b are positive integers, not necessarily unequal. If 36ab is divisible by 4, and also by 6, then how many possible values of "36ab" exist?
1] 6 2] 9 3] 8 4] 7

7. Fill in the missing digit in "53 _ 3559575" so that the number is a multiple of 11.
1] 3 2] 1 3] 7 4] 6

Answer Key

1-2 2-4 3-2 4-4 5-2 6-4 7-4

Explanatory Answers

1. (a) and (d)

2. 2181 – divisible by 3
3585 – divisible by 5
4587 – divisible by 11
3931 – prime number (check up to divisibility by prime numbers upto square root of 3931)

3. $11979 = 11^3 \times 9$. So it needs to be divided by 9.

4. ab has to be even and 109521ab divisible by 11. Using divisibility by 11, 6+b = 12 + a or b = 6 + a Answer is (1). 39 cannot fit the bill, as it is odd.

5. There are 101 numbers between 100 and 200, both inclusive (starting with an even number).
$\therefore$ 51 of these numbers are even, i.e. divisible by 2.
Again, the numbers divisible by 3 start from 102 (34×3) and end at 198 (66×3).
From 34 to 66, there are 33 numbers.
Similarly, as $16 \times 6 = 96$, there are sixteen numbers among those 101 that are divisible by both 2 and 3.
$\therefore$ The required number = 51 + 33 – 16 = 68.

6. For a number N to be divisible by 4 and 6, N must be divisible by LCM of (4, 6). Now 3600 is divisible by 12.
The possible numbers are 3612, 3624, 3636,... 3696. (i.e. 3600 + 12 × 1, 3600 + 12 × 2, …3600 + 12 × 8).
Total = 7 numbers.
Note: 3600 is divisible by 4 and 6, but 0 is neither positive nor negative.
The number 3660 is not possible, because in "36ab", 'a' and 'b' are positive integers

7. For a number to be divisible by 11, the difference of the sum of digits in even position and sum of the digits in odd position must be 0 or a multiple of 11.
In the number 53 x 3559575
Sum of digits in odd position = 5 + 5 + 5 + 3 + 3 = 21
Sum of digits in even position = 7 + 9 + 5 + x + 5 = 26 + x
$\therefore 26 + x - 21 = 11 \Rightarrow 5 + x = 11 \Rightarrow x = 6$
$\therefore$ The number is 536 355 9575. Hence [4]

Practice Questions B (Optional)

1. Which of the following is divisible by 11?
1] 123456789 2] 123454321 3] 112233221 4] None of the above

2. The number x ! (x factorial) is definitely
1] divisible by 6
2] divisible by 3
3] divisible by 6 if x > 2
4] not divisible by 6 if x < 5

3. How many 3-digit numbers of the form 36X are divisible by 3?
1] 2 2] 3 3] 4 4] 1

4. How many 4-digit numbers of the form 36AB are divisible by 15?
1] 6 2] 8 3] 7 4] 9

5. If 8a765 + 8765a (where 'a' is an integer between 0... 9)is divisible by 9 then what is 'a'?
1] 1 2] 3 3] 2 4] 4

6. Let N = Product of any three consecutive natural numbers. Then N is certainly divisible by
1] 6 2] 4 3] 5 4] 7

7. What are the digits a,b so that the number a759b is divisible by 72?
1] 4,2 2] 2,4 3] 3,6 4] 6,3

8. Let $N = 2^{2n} - 1$. Then which of the following statements are true?

i. N is always divisible by 3
ii. N is always divisible by 3 and 5
iii. N is divisible by 5 if n is odd
iv. N is divisible by 5 if n is even

1] i and iii 2] ii only 3] i and iv 4] iii and iv

Answer Key

1-4 2-3 3-3 4-3 5-1 6-1 7-1 8-3

Chapter 3 Power Cycles

Suggested Time : 30 minutes

Look very closely at the following values

2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}
2	4	8	16	32	64	128	256	512	1024	2048	4096

The unit's place of the numbers show a trend. They are 2, 4, 8, 6, 2, 4, 8, 6, …. In other words, the units place repeats in cycles of 4. Now try the same for other numbers like 3, 4, 5, 6, 7, …

Power	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8
Unit's Digit of powers when x = 3	3	9	7	1	3	9	7	1
Unit's Digit of powers when x = 4	4	6	4	6	4	6	4	6
Unit's Digit of powers when x = 5	5	5	5	5	5	5	5	5
Unit's Digit of powers when x = 6	6	6	6	6	6	6	6	6
Unit's Digit of powers when x = 7	7	9	3	1	7	9	3	1
Unit's Digit of powers when x = 8	8	4	2	6	8	4	2	6
Unit's Digit of powers when x = 9	9	1	9	1	9	1	9	1
Unit's Digit of powers when x = 12 (same as 2)	2	4	8	6	2	4	8	6
Unit's Digit of powers when x = 13 (same as 3)	3	9	7	1	3	9	7	1

We can say that the unit's digit in powers of the number 3 shows the trend 3, 9, 7, 1, …, i.e. a cycle of 4. In other words every 4th power is 1. Or alternately, 3^{4n} ends in 1. Hence, all numbers of the form 3^{4n} end in 1, 3^{4n+1} end in 3, 3^{4n+2} end in 9 and 3^{4n+3} end in 7.

We can say that the unit's digit in powers of the number 4 shows the trend 4, 6, 4, 6, …i.e. a cycle of 2. However, a cycle of 2 also means a cycle of 4. In other words every 4th power is 6. Or alternately, 4^{4n} ends in 6. Hence, all numbers of the form 4^{4n} end in 6, 4^{4n+1} end in 4, 4^{4n+2} end in 6 and 4^{4n+3} end in 4.

This is true for all numbers, i.e. **For all numbers, the unit's digit repeats in a cycle of 4**.

Hence, $13^{13} = 13^{12+1} = 13^{4n+1}$. Now 3 shows a cycle of 3, 9, 7, 1…. So 3^{4n} ends in 1 and 3^{4n+1} end in 3. So 13^{13} ends in 3, or in other words it has 3 in the unit's place.

Practice Questions A (Optional)

1. What is the digit in the unit's place in the expansion of 3^{83}?
1] 3 2] 9 3] 7 4] 1

2. What digit comes in the unit's place if 12^{12} is multiplied by 13^{13}?
1] 6 2] 8 3] 2 4] 4

3. What is the last (unit's place) digit in the expansion of 17^{286}?
1] 9 2] 3 3] 1 4] 7

4. What is the last (unit's place) digit in the expansion of $17^{28} \times 19^{86}$?
1] 9 2] 3 3] 1 4] 7

Answer Key

1-3 2-2 3-1 4-4

Explanatory Answers

1. $3^{83} = 3^{80+3} = 3^{4 \times 20 + 3} = 3^{4k+3}$.
Power cycle of 3 is 3, 9, 7, 1, …, i.e. 3^{4k} ends in 1
Hence, 3^{4k+3} ends is 7.
Hence [3]

2. $12^{12} = 12^{4k}$ and $13^{13} = 13^{4k+1}$
Power cycle of 2 is 2, 4, 8, 6, … and power cycle of 3 is 3, 9, 7, 1, …
Hence, 12^{12} ends in 6 and 13^{13} ends in 3.
$\therefore 12^{12} \times 13^{13}$ ends in 6 × 3 = 18, i.e. the number ends in 8.
Hence [2]

3. 17 will have the same power cycle as 7.
$7^1 = 7$
$7^2 = 7 \times 7 = 49$
$7^3 = 49 \times 7 = 343$
$7^4 = 343 \times 7$ ... this has 1 at the unit's place
$\therefore$ Obviously, 7^5 will have $1 \times 7 = 7$ in the unit's place.
Thus, powers of 7 have 7, 9, 3, and 1 occurring simultaneously at the unit's place.
$286 = 71 \times 4 + 2$
$\therefore 17^{285}$ will have 7 at the unit's place.
$\therefore 17^{286}$ will have 9 at the unit's place. Hence, (1).

4. 17 will have the same power cycle as 7. Thus, powers of 7 have 7, 9, 3, and 1 occurring simultaneously at the unit's place.
$17^{28} = 17^{4k}$. Hence 17^{28} will end with 1.
19 will have the same power cycle as 9. Thus, powers of 9 have 9, 1, 9, and 1 occurring simultaneously at the unit's place.
$19^{86} = 19^{4k+2}$. Hence 19^{86} will end with 1.
Hence $17^{28} \times 19^{86}$ will be written as ___1 × ___1, i.e. It will be _______1. i.e. end with 1.

Chapter 4 Highest Power

Suggested Time : 45 minutes

A less popular but an easy type of question: Let us take an example. 5! is 120.

Note : 5! means 5 factorial. A factorial of any number ‘n’ is the value of the expression n × (n – 1) × (n – 2) × (n – 3) ×…× 1. Hence, 5! = 5 × 4 × 3 × 2 × 1 = 120

Is 5! divisible by 2? Yes.
Is 5! divisible by 4? Yes.
Is 5! divisible by 8? Yes.
Is 5! divisible by 16? No.

5! is divisible by 2^1, 2^2 and 2^3 but not 2^4. So the highest power of 2 that divides 5! is 3.

Alternately, we can try and divide all the numbers in the expansion of 5! by 2.
1 is not divisible by 2.
2 is divisible by 2.
3 is not divisible by 2.
4 is divisible by 2, and the quotient is 2, which is further divisible by 2.
5 is not divisible by 2.
So 2 and 4 are the only numbers that are divisible by 2, and the highest power of 2 that divides (2 × 4) is 2^3.

Illustration

What is the largest possible value of ‘x’ in the following case, given that the numerator is perfectly divisible by the denominator?

4. $\frac{10!}{2^x}$

1] 5 2] 6 3] 7 4] 8

10! is 10 × 9 × … × 2 × 1
Of these, only 2, 4, 6, 8 and 10 are divisible by 2.
When 2 is divided by 2, the quotient is 1.
When 4 is divided by 2, the quotient is 2, which is further divisible by 2.
When 6 is divided by 2, the quotient is 3.
When 8 is divided by 2, the quotient is 4. This is again divided by 2, and the quotient is 2, which is further divisible by 2.
When 10 is divided by 2, the quotient is 5.

In other words, 2, 6 and 10 are divisible once, 4 is divisible twice and 8 is divisible thrice by 2. So 10! is divisible 1 + 1 + 1 + 2 + 3 = 8 times by 2. So the highest power of 2 that divides 10! is 8.

Googly Question

What is the highest power of 6 that divides 10!?
The answer is not 1. It is 4.

Note that 6 is 2 × 3. Hence, we should follow the following steps
1. What is the highest power of 2 that divides 10!? $\Rightarrow$ 8
2. What is the highest power of 3 that divides 10!? $\Rightarrow$ 4
3. What is the highest power of 6 that divides 10!?

From steps 1 and 2 we know that 10! is divisible by $2^8 \times 3^4$. Alternately, 10! is divisible by $2^4 \times (2^4 \times 3^4)$. Hence, 10! is divisible by $2^4 \times 6^4$. So the highest power of 6 that divides 10! is 4.

Shortcut 1

What is the largest possible value of 'x' in the following case, given that the numerator is perfectly divisible by the denominator?

$$\frac{10!}{3^x}$$

Upto 10, the number of numbers divisible by 3^1 are 3. (only 3, 6, 9)
Upto 10, the number of numbers divisible by 3^2 are 1. (only 9)
Hence, answer is 3 + 1 = 4

Shortcut 2

What is the largest possible value of 'x' in the following case, given that the numerator is perfectly divisible by the denominator?

$$\frac{10!}{6^x}$$

Note : Dividing by 6^x is dividing by $2^x \times 3^x$. Every 2nd number is divisible by 2, but every 3rd number is divisible by 3. Hence, there are lesser numbers that are divisible by 3.

If $6^x = 2^a \times 3^b$, then b < a. This means that we need to only find out the highest power of 3 that divides 6!, as the number of numbers in 6! that are divisible by 2 are more.

i.e. The question is the same as $\frac{10!}{3^x}$

Now, divide 10 by 3 and write the quotient, and then repeat this exercise by dividing the quotient by 3, and so on, i.e. $\frac{10}{3} = \frac{3}{3} = 1$.

Finally, add all the quotients you get in this process. In the above example it is 3 + 1 = 4.

Extension of shortcut 2

Another way of expressing the above is

$$\begin{array}{r|l} 3 & 10 \\ \hline 3 & 3 \leftarrow \\ \hline & 1 \leftarrow \end{array}$$

Taking the sum of the quotients we get 3 + 1 = 4.

Practice Questions A (Optional)

1. How many of the following numbers divide 5! (i.e. 5 factorial)?

(a) 3 (b) 5 (c) 10 (d) 15 (e) 16

1] 2 2] 3 3] 4 4] 5

Directions for questions 4-7

What is the largest possible value of 'x' in each of the following cases, given that in each case the numerator is perfectly divisible by the denominator?

2. $\frac{10!}{2^x}$

1] 5 2] 6 3] 7 4] 8

3. $\frac{10!}{3^x}$

1] 2 2] 3 3] 4 4] 5

4. $\frac{10!}{6^x}$

1] 1 2] 2 3] 3 4] 4

5. $\frac{10!}{10^x}$

1] 1 2] 2 3] 4 4] 5

6. What is the highest power of 4 that can divide 40!?

1] 19 2] 40 3] 12 4] 38

Answer Key

1-3 2-4 3-3 4-4 5-2 6-1

Explanatory Answers

1. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 5 \times 2 \times 2 \times 3 \times 2 \times 1 = 5 \times 2^3 \times 3$.

Hence 5! is divisible by 3, 5, 10 (i.e. 2×5), 15 (i.e. 3×5), but not 16 (i.e. 2^4). Hence [3]

2. Using extension of shortcut 2,

$$\begin{array}{r|l} 2 & 10 \\ \hline 2 & 5 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Adding the quotients, we get 5 + 2 + 1 = 8. Hence [4]

3. Using extension of shortcut 2,

$3\underline{|10}$

$3\underline{|3}$ Adding the quotients, we get 3 + 1 = 4. Hence [3]

$\;\;1$

4. For 6, we divide by 3 as 6 = 2 × 3, and the constraining factor is 3.

We get $\frac{10}{3}=\frac{3}{3}=1$. Hence answer = 3 + 1 = 4. Hence [4]

5. For 10, we divide by 5 as 10 = 2 × 5, and the constraining factor is 5.

We get $\frac{10}{5}=2$. Hence answer = 2. Hence [2]

6. $\frac{40!}{4^x}=\frac{40!}{(2\times 2)^x}$

$2\underline{|40}$

$2\underline{|20}$

$2\underline{|10}$

$2\underline{|5}$

$2\underline{|2}$

$\;\;1$

Highest power of 2 that divides 40 ! is 20 + 10 + 5 + 2 + 1 = 38, i.e. $\frac{40!}{2^{38}}$

$\therefore$ Highest power of 2×2 that divides $40!$ is $\frac{38}{2}$ = 19, i.e. $\frac{40!}{\left(2^2\right)^{19}}$. Hence [1]

Chapter 5 # Factors **Suggested Time : 45 minutes**

Factor: A factor is a number which divides another number. For example, the factors of 10 are 1, 2, 5 and 10.

NUMBER OF FACTORS OF A GIVEN NUMBER

If N is a composite number such that N = $a^x b^y c^z$ … where a, b, c are prime factors of N and x, y, z are positive integers, then

The number of factors of N = (1 + x) (1 + y) (1 + z)…

SUM OF FACTORS

If N is a composite number such that N = $a^x b^y c^z$ … where a, b, c are prime factors and x, y, z are positive integers, then

The sum of the factors of N = $\left(\frac{a^{x+1}-1}{a-1}\right)\left(\frac{b^{y+1}-1}{b-1}\right)\left(\frac{c^{z+1}-1}{c-1}\right)\ldots$

Example:

Find out the sum of the factors of 200

N = 200 = $2^3 \times 5^2$

The sum of the factors = $\left(\frac{2^{3+1}-1}{2-1}\right)\left(\frac{5^{2+1}-1}{5-1}\right) = 15 \times 31 = 465$

SOME IMPORTANT RESULTS ON FACTORS

If N is a composite number such that N = $a^x b^y c^z$ … where a, b, c are prime factors and x, y, z are positive integers, then

1. $\left.\begin{array}{l}\textbf{The number of ways of expressing N}\\ \textbf{as product of two different factors}\end{array}\right\} = \frac{(1+x)(1+y)(1+z)\ldots}{2} = \frac{\text{Number of factors}}{2}$

2. If N is a perfect square, then

$$\left.\begin{array}{l}\textbf{The number of ways of expressing N}\\ \textbf{as product of two different factors}\end{array}\right\} = \frac{\{[(1+x)(1+y)(1+z)\ldots]-1\}}{2}$$

Note: All perfect squares (and only perfect squares) have odd number of factors.
Example: Number of factors of 36 is 9.
36 = 4 × 9
36 = $2^2 \times 3^2$
Number of factors = (2 + 1) × (2 + 1) = 9

Practice Questions A (Optional)

1. How many rectangles with integral sides and with an area 224 m^2 are possible?
1] 10 2] 12 3] 6 4] 8

2. What is the number of positive integers that can divide 200 without remainder?
1] 10 2] 6 3] 12 4] 5

3. The number N has 144 factors including 1 and itself. What is the maximum and minimum possible number of prime factors that N can have?
1] 144 & 1 2] 6 & 1 3] 8 & 2 4] 4 & 2

4. Find the number of positive integers, which divide 10^{999} but not 10^{998}.
1] 1999 2] 999 3] 2999 4] 1799

Answer Key

1-3 2-3 3-2 4-1

Explanatory Answer

1. To find out the number of rectangles with integral sides, we need to find out the number of ways of expressing 224 as a product of two factors.

$24 = 8 \times 28 = 8 \times 4 \times 7 = 2^5 \times 7^1$

Number of ways of expressing 224 as a product of two factors (OR)

Number of rectangles possible = $\frac{(1+5)(1+1)}{2} = 6$

2. If N = $a^x b^y c^z$ … where a, b, c are prime factors of N and x, y, z are positive integers, then
The number of factors of N = (1 + x) (1 + y) (1 + z)…
200 = $2^3 \times 5^2$
The number of factors = (3 + 1) (2 + 1) = 4× 3 = 12

3. If N = $a^x b^y c^z$ … where a, b, c are prime factors of N and x, y, z are positive integers, then
The number of factors of N = (1 + x) (1 + y) (1 + z)…
Given that the number of factors is 144 =$2 \times 2 \times 2 \times 2 \times 3 \times 3$
If the number N should be expressed with maximum prime factors then N = $a \times b \times c \times d \times e^2 \times f^2$ whose number of factors = $2 \times 2 \times 2 \times 2 \times 3 \times 3$ = 144.
∴ The maximum possible prime factors of N = 6
If the number N should be expressed with minimum prime factors then N = a^{143} whose number of factors = 143 + 1 = 144.
∴ The minimum possible prime factor of N = 1

4. If N = $a^x b^y c^z$ … where a, b, c are prime factors of N and x, y, z are positive integers, then
The number of factors of N = (1 + x)(1 + y) (1 + z)…

Let $N_1 = 10^{999} = 5^{999} \times 2^{999}$

∴Number of divisors of N = (999 + 1) (999 + 1) = 1000^2

Let $N_2 = 10^{998} = 2^{998} \times 5^{998}$

∴Number of divisors = (998 + 1) (998 + 1) = 999^2

∴ The number of positive integers, which divide 10^{999} but not 10^{998} is $1000^2 - 999^2$
= (1000 + 999) (1000 – 999) = 1999. Hence [1]