FLUID MECHANICS

CONTENTS

PART A: NOTES

CHAPTER 1

FLUIDS AND THEIR PROPERTIES

1.1 INTRODUCTION

Fluid Mechanics, as the name indicates, is that branch of applied mechanics which is concerned with the statics and dynamics of liquids and gases. The analysis of the fluid is based upon the fundamental laws of applied mechanics which relates to the conservation of mass, energy and the force momentum equation.

There are, however, two major aspects of fluid mechanics which differ from the solid body mechanics:

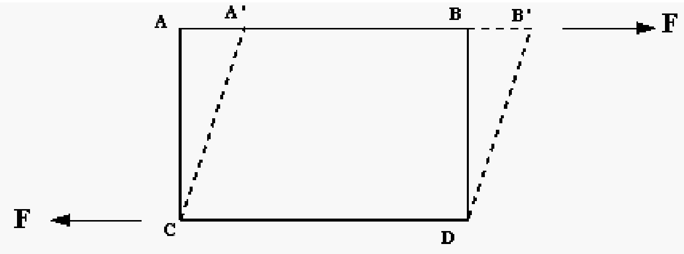
(i) The nature and properties of the fluid itself, which are very different from those of a solid.

(ii) Instead of dealing with individual bodies or elements of known mass, we are frequently concerned with the behavior of a continuous stream of fluid, without beginning or end.

1.2 FLUIDS

We normally recognize three states of matter: solid; liquid and gas. However, liquid and gas are both fluids: in contrast to solids they lack the ability to resist deformation. Because a fluid cannot resist the deformation force, it moves, it flows under the action of the force. Its shape will change continuously as long as the force is applied. A solid can resist a deformation force while at rest, this force may cause some displacement but the solid does not continue to move indefinitely.

The deformation is caused by shearing forces which act tangentially to a surface. Referring to the figure below, we see the force F acting tangentially on a rectangular (solid lined) element ABDC. This is a shearing force and produces the (dashed lined) rhombus element A'B'DC.



Shearing force, F, acting on a fluid element.

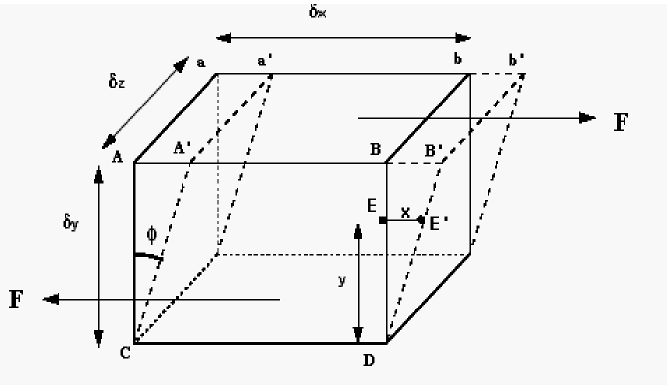
We can say

"A Fluid is a substance which deforms continuously or flows, when subjected to shearing forces."

and conversely this definition implies the very important point that:

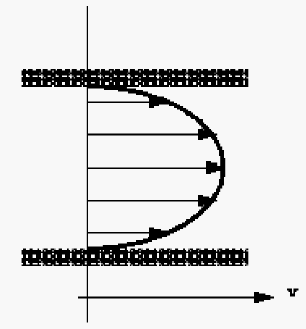
"If a fluid is at rest there are no shearing forces acting. All forces must be perpendicular to the planes which they are acting."

When a fluid is in motion shear stresses are developed if the particles of the fluid move relative to one another. When this happens adjacent particles have different velocities. If fluid velocity is the same at every point then there is no shear stress produced: the particles have *zero relative velocity*.



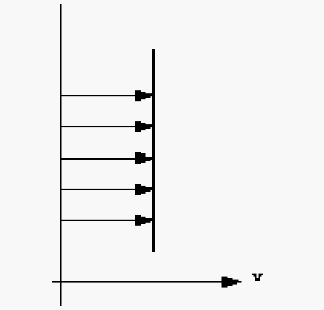
Fluid element under a shear force

Consider the flow in a pipe in which water is flowing. At the pipe wall the velocity of the water will be zero. The velocity will increase as we move toward the centre of the pipe. This change in velocity across the direction of flow is known as velocity profile and shown graphically in the figure below:



Velocity profile in a pipe.

Because particles of fluid next to each other are moving with different velocities there are shear forces in the moving fluid i.e. shear forces are **normally** present in a moving fluid. On the other hand, if a fluid is a long way from the boundary and all the particles are travelling with the same velocity, the velocity profile would look something like this:



Velocity profile in uniform flow

And there will be no shear forces present as all particles have zero relative velocity. In practice we are concerned with flow past solid boundaries; aero planes, cars, pipe walls, river channels etc. and shear forces will be present.

1.3 NEWTON'S LAW OF VISCOSITY

Let consider a 3d rectangular element of fluid, like that in the figure below.

The shearing force F acts on the area on the top of the element. This area is given by $A = \delta_s \times \delta_x$. We can thus calculate the *shear stress* which is equal to force per unit area i.e.

$$shear\ stress,\ \tau = \frac{F}{A}$$

The deformation which this shear stress causes is measured by the size of the angle Φ and is known as *shear strain.*

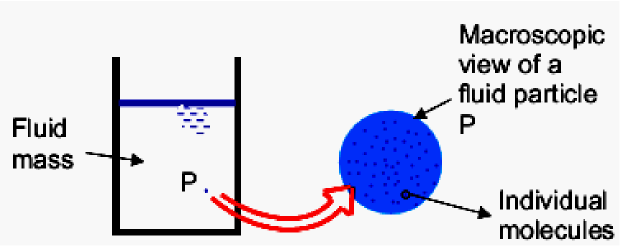
It has been found experimentally that the *rate of shear stress* (shear stress per unit time, τ / time) is directly proportional to the *shear stress*. If the particle at point *E* (in the above figure) moves under the shear stress to point *E'* and it takes time *t* to get there, it has moved the distance *x*. For small deformations we can write

$$shear\ strain,\ \phi = \frac{x}{y}$$

$$\begin{aligned} rate\ of\ shear\ strain\ &= \frac{\phi}{t} \\ &= \frac{x}{ty} = \frac{x}{t}.\frac{1}{y} \\ &= \frac{u}{y} \end{aligned}$$

Where $\frac{x}{t} = u$ the velocity of the particle at E.

Using the experimental result that shear stress is proportional to rate of shear strain then is



$$\tau = constant \times \frac{u}{y}$$

The term $\frac{u}{y}$ is the change in velocity with *y*, or the velocity gradient, and may be written in the differential form $\frac{du}{dy}$. The constant of proportionality is known as the dynamic viscosity, μ, of the fluid, giving $\boxed{\tau = \mu \times \frac{du}{dy}}$

This is known as *Newton's law of viscosity.*

1.4 THE CONTINUUM CONCEPT OF A FLUID

The behavior of individual molecules comprising a fluid determines the observed properties of the fluid and for an absolutely complete analysis; the fluid should be studied at the molecular scale. The behavior of any one molecule is highly complex, continuously varying and May indeed is very different from neighboring molecules at any instant of time. The problems normally encountered by engineers do not require knowledge and prediction of behavior at the molecular level but on the properties of the fluid mass that may result. Thus the interest is more on the average rather than the individual responses of the molecules comprising the fluid. At a microscopic level, a fluid consists of molecules with a lot of space in between. For our analysis, we do not consider the actual conglomeration of separate molecules, but instead assume that the fluid is a continuum, that is a continuous distribution of matter with no empty space. The sketch below illustrates this. However, we are interested in the property of the fluid particle at P and therefore we regard P as being a "smear" of matter (represented as a solid filled circle in the figure) with no space.

1.5 TYPES OF FLUID

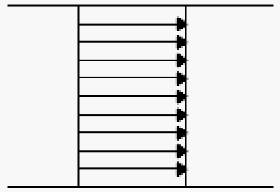
A fluid may be classified as follows:

1.5.1 IDEAL FLUID

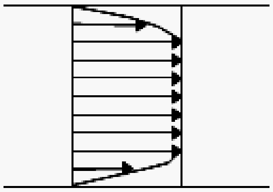
An ideal fluid does not possess properties like viscosity, surface tension and compressibility. Such a fluid will not offer any resistance to displacement of surfaces in contact that is $\tau = 0$. Ideal fluid is only an imaginary fluid.

1.5.2 REAL FLUID

A real fluid possesses viscosity, surface tension and compressibility. Such a fluid will always resist displacement that is $\tau \neq 0$. Real fluid is an actual fluid which exists.



Ideal flow



Real flow

1.5.3 COMPRESSIBLE – INCOMPRESSIBLE FLUID

If the volume of an element of fluid is independent of its pressure and temperature, the fluid is said to be *incompressible*.

If the volume of an element of fluid changes with its pressure and temperature, the fluid is said to be *compressible*.

Although, all fluids are compressible - even water - their density will change as pressure changes. Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density. In practical problems, the liquid are assumed to be incompressible and gases are said to be compressible.

1.6 PHYSICAL PROPERTIES

The most common properties of fluids are as follows:

1.6.1 DENSITY

The density of a substance is the quantity of matter contained in a unit volume of the substance. It can be expressed in three different ways.

1.6.1.1 MASS DENSITY (ρ)

Mass Density is defined as the mass of substance per unit volume.

Units: kg / m^3

Dimensions: ML^{-3}

Typical values:

Water = 1000 $kg\ m^{-3}$, Mercury = 13546 $kg\ m^{-3}$, Air = 1.23 $kg\ m^{-3}$.

(at P =$1.01325 \times 10^{-5}\ N\ m^{-2}$, T = 288.15 K)

1.6.1.2 SPECIFIC WEIGHT (w)

Specific Weight is defined as the weight per unit volume.

or

The force exerted by gravity, g, upon a unit volume of the substance.

The Relationship between g and ω can be determined by Newton's 2nd Law, since

Weight per unit volume = mass per unit volume × g

$$w = \rho . g$$

Units: N / m^3

Dimensions: $ML^{-2}T^{-2}$.

Typical values:

Water = 9814 $N\ m^{-3}$, Mercury = 132943 $N\ m^{-3}$, Air = 12.07 $N\ m^{-3}$.

1.6.1.3 SPECIFIC GRAVITY (*s*)

Specific Gravity is defined as the ratio of mass density of a substance to some standard mass density.

For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4 ^0C) at atmospheric pressure.

$$s = \frac{\rho}{\rho_w}$$

Units: None, since a ratio is a pure number.

Dimensions: 1.

Typical values: Water = 1, Mercury = 13.5, Air = 1.

1.6.2 VISCOSITY

Viscosity is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to sheer deformation. Different fluids deform at different rates under the same shear stress.

Fluid with a high viscosity such as syrup deforms more slowly than fluid with a low viscosity such as water.

All fluids are viscous; “Newtonian Fluids” obey the linear relationship, given by Newton’s law of viscosity. $\tau = \mu \frac{du}{dy}$.

Where τ is the shear stress; has

Units: $N\ m^{-2}$; $kg\ m^{-1}\ s^{-2}$

Dimensions: $ML^{-1}\ T^{-2}$

$\frac{du}{dy}$ is the velocity gradient or rate of shear strain, and has

Units: *radian* s^{-1}

Dimensions: T^{-1}

μ is the “coefficient of dynamic viscosity”

1.6.2.1 COEFFICIENT OF VISCOSITY (μ)

The Coefficient of Dynamic Viscosity is defined as the shear force, per unit area, (or shear stress τ), required to drag one layer of fluid with unit velocity past another layer a unit distance away.

$$\mu = \tau \Big/ \frac{du}{dy} = \frac{Force}{Area} \Big/ \frac{Velocity}{Distance} = \frac{Force \times Time}{Area} = \frac{Mass}{Area \times Length}$$

Units: $N\,s\,m^{-2}$ or, $kg\,m^{-1}\,s^{-1}$

Typical values:

Water = 1.14×10^{-3} kg m^{-1} s^{-1}, Air =1.78×10^{-5} kg m^{-1} s^{-1},

Mercury =1.552 kg m^{-1} s^{-1}

1.6.2.2 KINEMATIC VISCOSITY (υ)

Kinematic Viscosity is defined as the ratio of dynamic viscosity to mass density.

$$\upsilon = \frac{\mu}{\rho}$$

Units: $m^2\,s^{-1}$(υ is often expressed in *Stokes*, St, where 10^4 St = 1 $m^2\,s^{-1}$)

Dimensions: $L^2\,T^{-1}$.

Typical values:

Water =1.14×10^{-6} m^2 s^{-1}, Air =1.46×10^{-5} m^2 s^{-1}, Mercury =1.145×10^{-4} m^2 s^{-1}

1.6.3 SURFACE TENSION (σ)

Surface tension is defined as a tension force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquid, such that the contact surface behaves like a membrane under tension. It is the tendency of the surface of a liquid to behave like a stretched elastic membrane. Units: $N\,m^{-1}$

Dimensions: $M\,T^{-2}$

The forces of attraction binding molecules to one another give rise to *cohesion*, the tendency of the liquid to remain as one assemblage of particles rather than to behave as a gas and fill the entire space within which it is confined.

On the other hand, forces between the molecules of a fluid and the molecules of a solid boundary give rise to *adhesion* between the fluid and the boundary.

It is the interplay of these two forces that determine whether the liquid will *"wet"* the solid surface of the container.

- If the adhesive forces are greater than the cohesive forces, then the liquid will wet the surface.
- if the cohesive forces are greater than the adhesive forces, then the liquid will not wet the surface.

1.6.3.1 CAPILLARY

Capillarity phenomenon is defined as rise and fall of the liquid surface in a tube relative to the adjacent general level of liquid depends upon the specific weight of the liquid, diameter of tube and surface tension.

The water column in the sketch below rises to a height *h* such that the weight of the column is balanced by the resultant surface tension forces acting at θ to the vertical at the contact with the tube.

And from equilibrium of forces $\boxed{h = \dfrac{4\sigma Cos\theta}{\rho g d}}$

1.6.4 VAPOR PRESSURE

At the surface of a liquid, molecules are leaving and re-entering the liquid mass. The activity of the molecules at the surface creates a vapor pressure, which is a measure of the rate at which the molecules leave the surface. When the vapor pressure of the liquid is equal to the partial pressure of the molecules from the liquid which are in the gas above the surface, the number of molecules leaving is equal to the number entering. At this equilibrium condition, the vapor pressure is known as the saturation pressure.

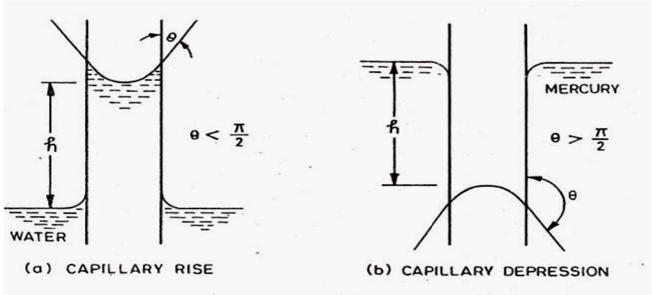
Units: $N\ m^{-2}$ (same as atmospheric pressure)

Dimensions: $ML^{-1}T^{-2}$.

Typical values:

Water = 760 mm Hg = 1.01325 bar = 1 atm

(at T = 100 ^{0}C)



(a) CAPILLARY RISE (b) CAPILLARY DEPRESSION

1.6.5 COMPRESSIBILITY AND BULK MODULUS (K)

The degree of compressibility of a substance is characterized by the *bulk modulus of elasticity*.

$$\boxed{K = -\frac{\Delta P}{\Delta V / V}}$$

Where, ΔP represents the small increased in pressure applied to the substance that causes a decrease of the volume by ΔV from its original volume of V.

The negative sign in the definition to ensure that the value of K is always positive.

Units: $N\,m^{-2}$

Dimensions: $ML^{-1}\,T^{-2}$.

Typical values:

For Water = 2.05×10^{9} N m^{-2}, For Air = 1.62×10^{9} N m^{-2}

1.7 DIFFERENCE BETWEEN SOLIDS AND FLUIDS

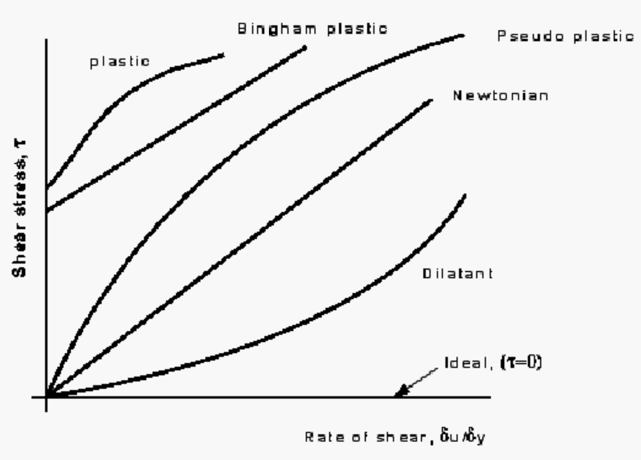
The differences between the behaviors of solids and fluids under an applied force are as follows:

i. For a solid, the strain is a function of the applied stress, providing that the elastic limit is not exceeded. For a fluid, the rate of strain is proportional to the applied stress.

ii. The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow as long as the force is applied and will not recover its original form when the force is removed.

1.8 TYPES OF FLUID BEHAVIOR

When the measured values of shear stress or viscosity are plotted versus shear rate, various types of behavior may be observed depending upon the fluid properties, as shown in figure



Shear stress vs. Rate of shear strain δu/δy

1.8.1 NEWTONIAN FLUIDS

If the shear stress versus shear rate plot is a straight line through the origin (or a straight line with a slope of unity on a log-log plot), the fluid is said to be Newtonian:

$$\tau = \mu \frac{du}{dy}$$

Where μ is the coefficient of viscosity.

1.8.2 NON NEWTONIAN FLUIDS

If the shear stress versus shear rate plot is not a straight line, the fluid is said to be Non Newtonian:

$$\tau = A + B\left(\frac{du}{dy}\right)^n$$

Where A, B and n are constants. For Newtonian fluids A = 0, B = μ and n = 1.

1.8.3 PLASTIC FLUIDS

A fluid, in which the shear stress is more than the yield value is known as the plastic fluid. i.e. Shear stress must reach a certain minimum before flow commences.

1.8.4 DILATANT FLUIDS

The fluids having no minimum shear stress and the Viscosity increases with rate of shear, and the value of n>1. e.g. *quicksand.*

1.8.5 BINGHAM PLASTIC FLUIDS

A minimum value of shear stress must be achieved before the fluid may start flow classification *n = 1*. An example is *sewage sludge.*

1.8.6 PSEUDO PLASTIC FLUIDS

The fluids having no minimum shear stress and the viscosity decreases with rate of shear and the value of n < 1, e.g. *colloidal substances like clay, milk and cement.*

1.9 TIME DEPENDENT FLUID

There are some fluids in which the viscosity depends upon the duration of shear. They can be classified as follows:

1.9.1 THIXOTROPIC FLUIDS

The fluids in which viscosity decreases with time for which shear force is applied e.g. *jelly paints.*

1.9.2 RHEOPECTIC FLUIDS

The fluids in which viscosity increases with time for which shear force is applied e.g. *gypsum suspension in water*

1.10 TEMPERATURE DEPENDENCY OF VISCOSITY

The viscosity of liquids and gases are varies with temperature as follows:

1.10.1 LIQUIDS

The viscosity of liquids varies with temperature as follows:

$$\mu = A\exp(B/T)$$

1.10.2 GASES

The viscosity of gases varies with temperature as follows:

$$\mu = AT^B$$

Where, A and B are the constants and T is the temperature.

Example 1.1 Calculate the specific weight, specific volume and specific gravity of a liquid having volume of 6 m^3 and weight of 44 kN.

Solution:

Specific weight, w :

$$w = \frac{\text{Weight of liquid}}{\text{Volume of liquid}} = \frac{44}{6} = 7.333\ kN/m^3$$

Specific mass, ρ :

$$\rho = \frac{w}{g} = \frac{7.333 \times 1000}{9.81} = 747.5\ kg/m^3$$

Specific Volume, υ :

$$\upsilon = \rho^{-1} = (747.5)^{-1} = 0.00134\ m^3/kg$$

Specific gravity, S :

$$S = \frac{w_{liquid}}{w_{water}} = \frac{7.333}{9.81} = 0.747$$

Example 1.2 The velocity distribution for flow over a flat plate is given by $u = 2y - y^2$ where u is the velocity in m / s at a distance y *meters* above the plate. Determine the velocity gradient and shear stress at the boundary and $1.5\ m$ from it.

Given Data: Dynamic viscosity of fluid is $0.9\ N.s / m^2$

Solution: $u = 2y - y^2$

$$\therefore \quad \frac{du}{dy} = 2 - 2y$$

- Velocity gradient, $\frac{du}{dy}$:

  At the boundary: At $y = 0, \left(\frac{du}{dy}\right)_{y=0} = 2s^{-1}$

  At 0.15 m from the oundary:

  $$\text{At } y = 0.15, \left(\frac{du}{dy}\right)_{y=0.15} = 1.7s^{-1}$$

- Shear stress, τ:

  At the boundary:

  $$(\tau)_{y=0} = \mu.\left(\frac{du}{dy}\right)_{y=0} = 0.9 \times 2 = 1.8\ N / m^2$$

  At 0.15 m from the oundary:

  $$(\tau)_{y=0.15} = \mu.\left(\frac{du}{dy}\right)_{y=0.15} = 0.9 \times 1.7 = 1.53\ N / m^2$$

Example 1.3 A $400\ mm$ diameter shaft is rotating at $200\ r.p.m.$ in a bearing of length $120\ mm$. if the thickness of oil film is $1.5\ mm$ and the dynamic viscosity of the oil is $0.7\ N.s / m^2$, determine:

(i) Torque required to overcome the friction in bearing;
(ii) Power utilizes in overcoming the viscous resistance.

Solution: Diameter of the shaft, $d = 400mm = 0.4m$

Speed of the shaft, $N = 200rpm$

Thickness of the oil film, $t = 1.5mm = 0.0015mm$

Length of the bearing, $l = 120mm = 0.12m$

Viscosity, $\mu = 0.7N.s / m^2$

Tangential velocity of the shaft

$$= \frac{\pi dN}{60} = \frac{\pi \times 0.4 \times 200}{60} = 4.19m / s$$

(*i*) Torque required, T :

We know, $\tau = \mu \frac{du}{dy}$

$$where\ du = \text{ change of velocity } = u - 0 = 4.19\ m/s$$

$$dy = t = 0.0015m$$

$$\therefore \qquad \tau = 0.7 \times \frac{4.19}{0.0015}$$

$$\tau = 1955.3\ N/m^2$$

Shear Force, $F = shear\ \ stress \times area$

$$F = \tau \times \pi dl$$

$$F = 1955.3 \times \pi \times 0.4 \times 0.12$$

$$F = 294.85\ N$$

$\Rightarrow$ Viscous torque $= F \times d/2$

$$= 294.85 \times 0.4/2 = \boxed{58.97\ N}$$

(*ii*) Power utilised, P :

$$P = T \times \frac{2\pi N}{60}, watts$$

$$P = 58.97 \times \frac{2\pi \times 200}{60} = \boxed{1.235\ kW}$$

Example 1.4 A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N / m. the angle of contact of the liquid with the glass can be assumed to be 135^0. The density of the liquid = 13600 kg / m^3. What would be the level of the level in the tube relative to the free surface of the liquid inside the tube?

Solution: $Given:\ d = 2.5\ mm;\ \sigma = 0.4/m,\ \theta = 135^0,\ \rho = 13600\ kg/m^3$

The liquid level in the tube rises (or falls) due to capillary. The capillary rise (or fall),

$$h = \frac{4\sigma Cos\theta}{wd}$$

$$h = \frac{4 \times 0.4 \times Cos135^0}{(9.81 \times 13600) \times 2.5 \times 10^{-3}}$$

$$\boxed{h = -3.39\ mm}$$